An Example

Look at the example below and two possible solutions.

As you read them, ask yourself whether you understand what the writer is doing, and whether you find it easy to follow.

Example Find an expression for the radius r of a circle of area A. Use it to find the radius, correct to 3 significant figures, when the area is 10 cm^2.

Solution 1

Area	πr^2
so	$r = \sqrt{A/\pi}$
	$A : 10$
	$r : \sqrt{10/\pi}$
	1.78

How easy did you find it to read this solution?

Were there any symbols you thought were wrong (like the colons:)?

Think about what you would do to make this more readable, then look at the next solution.

Solution 2

The area of a circle is given by $A = \pi r^2$, where r is the radius of the circle.

$$\begin{aligned} \text{So } r^2 &= A/\pi \\ \text{and hence } r &= \pm\sqrt{A/\pi}. \end{aligned}$$

We take the positive square root since r represents a length so $r = \sqrt{\dfrac{A}{\pi}}$.

When $A = 10, r = \sqrt{\dfrac{10}{\pi}} \approx 1.78$. Therefore when the area is 10 cm^2 the radius is approximately 1.78 cm.

This is much easier to read. Let's look at why this is. First of all it is written in sentences, with words joining up the mathematical expressions, which makes it much easier to read. Let's look at the solution again, and see what other additions make it clearer.

Solution 2(commented)

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Therefore when the area is 10 cm^2 the radius is approximately 1.78 cm.

In the solution above we used a linking word, namely "So" and "hence", at the start of some of the lines. In the next example the solution tries to use a symbol to link each line.

Look at the solution below, and see if you think this has been done correctly.

Example Turn the equation $t^2 = \dfrac{2(4-ut)}{9}$ round so that it reads

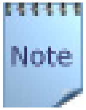
$$u = \text{some expression not involving } u.$$

(This is usually called changing the subject of an equation.)

Solution

$$\begin{aligned} t^2 &= \frac{2(4-ut)}{9} \\ = 9t^2 &= 2(4-ut) \\ = 9t^2 &= 8-2ut \\ = 2ut &= 8-9t^2 \\ = u &= \frac{8-9t^2}{2t} \end{aligned}$$

Is this solution written correctly? Well no it isn't. If you spotted why this was the case then click on the notebook icon to see if you were correct. If you didn't spot what was wrong with it then click on the notebook icon anyway.



Go to the next page to see what other symbols we could use instead.

Well, we could use the symbol $\Rightarrow$ meaning *implies* at the start of each line. This means that each line follows from the previous line. In this case, each of the lines is true if and only if the previous line is true, so we could use the stronger symbol $\Leftrightarrow$ meaning *if and only if*. The solution would then look like this, which is correctly written.

$$\begin{aligned}
t^2 &= \frac{2(4-ut)}{9} \\
\Leftrightarrow 9t^2 &= 2(4-ut) \\
\Leftrightarrow 9t^2 &= 8-2ut \\
\Leftrightarrow 2ut &= 8-9t^2 \\
\Leftrightarrow u &= \frac{8-9t^2}{2t}, \text{ assuming } t \neq 0.
\end{aligned}$$

An alternative way of presenting this argument would be to use link words as in the previous example. For instance the following would have been equally acceptable.

$$\begin{aligned} t^2 &= \frac{2(4-ut)}{9} \\ \text{means that } 9t^2 &= 2(4-ut) \\ \text{which gives } 9t^2 &= 8-2ut \\ \text{which can be written as } 2ut &= 8-9t^2. \\ \text{This gives } u &= \frac{8-9t^2}{2t}, \text{ assuming } t \neq 0. \end{aligned}$$

To finish, we give you two questions where you can write out the solution for yourself, then look at some of our comments.

Example Find the equation of the straight line joining the points $(2, 7)$ and $(5, 3)$.

Check your solution follows the writing guidelines and then go to the next page and compare your solution with ours.

Solution The equation of a straight line has the form $y = mx + c$.

The point (2,7) lies on the line, so substituting $x = 2$, $y = 7$ gives

$$\begin{aligned} 7 &= 2m + c. \quad (1) \\ \text{Similarly } (5,3) \text{ lies on the line so} \quad 3 &= 5m + c. \quad (2) \\ \text{Subtracting (1) from (2) gives} \quad -4 &= 3m \\ \text{so} \quad m &= -4/3. \end{aligned}$$

Substituting this value of m into equation (1) gives

$$\begin{aligned} 7 &= 2 \times -\frac{4}{3} + c \\ \text{or } c &= 7 + 8/3 = 29/3. \\ \text{Hence the required solution is } y &= -\frac{4}{3}x + \frac{29}{3} \\ \text{or } 3y &= -4x + 29. \end{aligned}$$

You may have done this by a different method, for instance you may know that the formula for a straight line through the points (x_1, y_1) and (x_2, y_2) is

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

and you may have used this formula instead.

Now try this example

Example

Find the cubic Taylor polynomial for $f(x) = (1 + x)^{1/2}$ about 0.

Look on the next page for the solution.

Solution The formula for the Taylor polynomial of degree 3 for $f(x)$ about 0 is

$$p(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3.$$

The first three derivatives of $(1+x)^{1/2}$ are

$$f'(x) = \frac{1}{2}(1+x)^{-\frac{1}{2}},\ f''(x) = -\frac{1}{4}(1+x)^{-\frac{3}{2}},\ \text{and}\ f^{(3)}(x) = \frac{3}{8}(1+x)^{-\frac{5}{2}}.$$

Evaluating f and its first three derivatives at 0 gives:

$$f(0) = 1,\ f'(0) = 1/2,\ f''(0) = -1/4,\ f^{(3)}(0) = 3/8.$$

So the Taylor polynomial of degree three about zero for $f(x) = (1+x)^{1/2}$ is

$$p(x) = 1 + \frac{1/2}{1!}x + \frac{(-1/4)}{2!}x^2 + \frac{3/8}{3!}x^3 = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}.$$

Hopefully your solutions are clear and readable and have the same properties as our solutions, namely

- The proof is written in sentences, so it is easy to read.
- Every step in the proof is explained, so you should be able to read it as easily some time in the future.
- Because every step is explained, even if you make a mistake in algebra your tutor could follow through what you had done, and give you "follow through" marks if you complete the rest of the problem correctly.

HAPPY WRITING!

End of main text. You can now close this page and return to the home page where you can choose to visit a different section.

Note

This solution is wrong because by putting = at the beginning of each line you are saying, for example, that all the expressions separated by = are equal to each other . So $t^2 = 9t^2$ and $2ut = 8 - 2ut$, but neither of these is true for all values of t and u.

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